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PROPERTIES OF LARGE AMPLITUDE LANGMUIR SOLITONS

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ABSTRACT

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I. INTRODUCTION

Over the past few years the soliton concept has emerged as one of the fundamental entities in the description of the physics of strongly interacting systems. In many areas of study (e.g., field theory, solid state, applied mathematics) the principal interest in solitons can be attributed to the remarkable elastic scattering properties of these objects. Recently, in plasma physics there has been an intensive effort devoted to the investigation of Langmuir solitons. While the scattering properties of such solitons in plasmas are of great theoretical interest, in practice this aspect has not been the major motivating reason for their study. The growing interest in the properties and generation of Langmuir solitons has emerged out of the realization that such entities constitute intense localized electric fields which can interact strongly with the plasma particles. Accordingly, the presence of such solitons in a plasma can modify the zero order quantities of the medium (density, temperature, flow velocity) in a significant manner.

In recent experiments 9-11 in which soliton-like structures in the Langmuir frequency range have been observed, two important points have been ellucidated:

1) the generation and behavior of the localized fields is intimately related to the presence of an external energy source (e.g., electron beam, quasi-electrostatic pump, electromagnetic radiation), 2) due to the external pumping, the energy density of these fields can become comparable to the kinetic energy density of the plasma; thus the associated density depressions or cavities can attain levels as large as 20 to 50 percent of the background density. Accordingly, these findings suggest two research topics which deserve considerable attention. Namely, the interaction and generation of Langmuir solitons in the presence of an external pump, and the study of such processes when the amplitude of the electric field is sufficiently large so that the usual cubic

nonlinearity description of the Langmuir soliton is not appropriate. The present study deals with these issues.

The present investigation aims at isolating within the simplest model some of the experimentally interesting properties associated with the generation, scattering, and response to external pumping, of large amplitude Langmuir solitons. Specifically, the mathematical model used consists of the nonlinear Schrodinger equation in which the full exponential nonlinearity is retained, i.e., the plasma density n is given by $n=n_0 \exp[-|E|^2/8\pi n_0 T_e]$ where, n_0 is the zero order plasma density, E refers to the high frequency electric field, and T_e is the electron temperature. Both, the homogeneous and inhomogeneous (i.e., with a source) versions of this equation are investigated numerically in an idealized one dimensional plasma slab. The identification of the principal features of this simple model are intended to aid and stimulate the search for localized fields in the Langmuir frequency range.

Some of the interesting properties discussed in the present report are: the steady state eigenvalues and wavefunctions of large amplitude Langmuir solitons and their relationship to small amplitude solitons, the time evolution of initial standing waves in a bounded slab into large amplitude solitons, the collisional properties of large amplitude solitons, the external pumping of stationary, moving, and colliding large amplitude solitons.

The manuscript is organized as follows: In Section II the mathematical formulation is presented. Section III discusses the steady state eigenvalues and large amplitude wavefunctions. In Section IV the time dependent phonomena are presented. Section IV A introduces the numerical method, Section IV B deals with the standing waves, Section IV C treats the collisional

properties, Section IV D is concerned with the pumping of a stationary soliton, Section IV E discusses the pumping of a moving soliton, and Section IV F treats the effect of external pumping during a collision. Conclusions are presented in Section V.

II. FORMULATION

In the present study one is interested in describing the space-time (x,t) evolution of the total high frequency electric field E, which is expected to exhibit oscillations in the Langmuir frequency range, ω_p . This field is described in terms of a modulational representation

$$E(x,t) = \tilde{E}(x,t)e^{-i\omega}p^{t} + c.c.$$
 (1)

where the complex amplitude \tilde{E} is assumed to vary slowly in time, i.e., $|\partial \tilde{E}/\partial t| << \omega_{p} |\tilde{E}|.$

The lowest order nonlinear description of the plasma can be obtained by using the two-fluid representation and separating in these equations the appropriate effects into a slow and a fast time scale. The evolution of the high frequency density n_h is determined n_h from

$$\frac{\partial^2}{\partial t^2} n_h + \frac{\partial}{\partial x} \left[-\frac{e n_{\ell}}{m} E - \frac{3 T_e}{m} \frac{\partial}{\partial x} n_h \right] = 0$$
 (2)

where n_{ℓ} refers to the slowly varying density, e and m represent the electron charge and mass, and T_{e} is the electron temperature. It should be noted that the dominant nonlinearity retained in Eq.(2) consists of the nonlinear modification of the slowly varying density n_{ℓ} , which must be determined self-consistently from other considerations. Furthermore, all the important single particle-field interactions (e.g. Landau damping, ion acceleration, plasma heating) are neglected.

The electric field in Eq.(2) is determined from Poisson's equation

$$\frac{\partial}{\partial x} E = -4\pi e n_h + 4\pi \rho \tag{3}$$

in which the possibility of having external charge sources ρ is allowed. The effect of these sources can equivalently be handled through the zero order vacuum field E which is created by ρ , i.e., Eq.(3) can be rewritten as

$$\frac{\partial}{\partial x} (E - E_0) = -4\pi e n_h \tag{4}$$

The form of Eq.(4) is useful in eliminating n_h out of the description, thus yielding from Eq.(2)

$$2i\omega_{p}\frac{\partial}{\partial t}\tilde{E} + \frac{3T_{e}}{m}\frac{\partial^{2}}{\partial x^{2}}\tilde{E} + (\omega_{p}^{2} - \frac{4\pi e^{2}n_{\ell}}{m})\tilde{E} = \omega_{p}^{2}\tilde{E}_{o}$$
 (5)

where the external field is assumed to have a dependence $E_0 = E_0 \exp(-i\omega_p t) + c.c.$ In arriving at Eq.(5) one makes explicit use of the slow time variation of \tilde{E} in order to neglect the second time derivative.

In general, the evolution of n_{ℓ} should be determined from the ion acoustic wave equation with the effects of the ponderomotive force included self-consistently¹². Due to the large amplitude levels which some of these fluctuations may attain, it is possible that even such an equation should be appropriately modified to include nonlinear distortions in the propagation characteristics, as has been investigated by Nishikawaka, et al.¹³. However, in the present study we wish to retain only the fundamental effect of nonlinear self-saturation of the ponderomotive force without introducing additional complications. For this reason we restrict this investigation to the effects associated with the simplified exponential nonlinearity mentioned in Sec. I. This admittedly oversimplified description can be visualized by representing the slow time evolution of the electron velocity \mathbf{v}_{ℓ} through

$$\frac{d}{dt} v_{\ell} \approx -\frac{\partial}{\partial x} \langle \frac{v_{h}^{2}}{2} \rangle$$
 (6)

where the brackets refer to a time average over the fast $2\pi/\omega_p$ scale, and v_h can be appropriately represented by the high frequency jitter velocity

$$\langle v_h^2 \rangle = (\frac{e}{m\omega_p})^2 |\tilde{E}|^2$$
 (7)

Using $\omega_p^{2=4\pi e^2} n_o/m$ gives Eq.(6) the form

$$m \frac{d}{dt} v_{\ell} = -\frac{\partial}{\partial x} \left(\frac{|\tilde{E}|^2}{8\pi n_0} \right)$$
 (8)

thus suggesting that the slow time electron motion is governed by the gradient of the slow pseudo-potential $\phi_p = |\tilde{E}|^2/8\pi n_o$. Accordingly, the low frequency electron density is expected to satisfy the Boltzmann expression

$$n_{\ell} = n_{0} \exp \left(-\phi_{p}/T_{e}\right) \tag{9}$$

which contains the important self-saturation effect which is absent in the usual cubic nonlinearity description of Langmuir solitons, i.e., the expansion of Eq.(9) in the limit $|\tilde{E}|^2/8\pi n_0 T_e <<1$. The underlying assumption connected with the use of Eq.(9) is that the inertial effects associated with the ions are negligible and that sufficient time elapses so that the ion and electron densities balance each other out so as to cancel the ambipolar potential.

An important point associated with the analytical description of n_{ℓ} through Eq.(9) is that sufficient care must be exercised in eliminating those spurious contributions contained in ϕ_p which do not cause changes in density (e.g., a traveling plane wave). The correct explicit version of Eq.(9) should read

$$n_{\hat{z}} = n_{o} \exp[-(|\tilde{E}|^{2} - \eta)/(8\pi n_{o}^{T} - \eta)]$$
 (10)

where η denotes the spatially uniform term which must be subtracted from $|\tilde{E}|^2 \text{ to cancel the spurious contributions.}$

When dealing with spatially periodic boundary conditions the quantity n can be identified with the k=0 component of the Fourier transform of $|\tilde{E}|^2$, and can be calculated by such a method. However, when one wishes to describe the generation of density cavities in a bounded slab, as is the case of interest in this study, such a procedure does not prove useful. In dealing with nonlinear density changes in a bounded model it must be realized that the total number of plasma particles is strictly conserved. This implies that if a cavity is formed at a certain position, then the density must increase at another location within the bounded slab. If the ion acoustic equation were to be used as a replacement for Eq.(9), then this physical effect would manifest itself automatically by the formation of density ridges or density bumps 12,14 surrounding the density cavities produced by the ponderomotive force. In order to incorporate this type of phenomenon into the description of n_{ℓ} through Eq.(10) one must determine the quantity n self-consistently by requiring that

$$\frac{\partial}{\partial t} \left[\int_{-k/2}^{k/2} dx n_k(x,t) \right] = 0$$
 (11)

where & refers to the length of the plasma slab. With this additional constraint the mathematical model of the problem is then complete.

The following scaled variables prove useful in describing the response of the system defined by Eqs.(5), (10), and (11)

$$\tau = \frac{\omega_{p}^{t}}{2} , \quad z = (\frac{m\omega_{p}^{2}}{3T_{e}})^{1/2} x$$

$$A = \frac{\tilde{E}}{(8\pi n_{o}^{T} e)^{1/2}} , \quad c = \frac{\eta}{(8\pi n_{o}^{T} e)^{1/2}}$$

$$p = \frac{E_{o}}{(8\pi n_{o}^{T} e)^{1/2}}$$
(12)

with these variables the model equation takes the form

$$i \frac{\partial}{\partial \tau} A + \frac{\partial^2}{\partial z^2} A + \left\{ 1 - \exp[-(|A|^2 - c)] \right\} A = p$$
 (13)

For an unbounded system and in the absence of external pumping c = p = 0. In such a limit one finds that Eq.(13) can be obtained from the Lagrangian density \mathcal{L} , where

$$\mathcal{L} = \left| \frac{\partial A}{\partial z} \right|^2 + \left[1 - \left| A \right|^2 - \exp\left(- \left| A \right|^2 \right) \right] + \frac{1}{2i} \left(A \star \frac{\partial}{\partial \tau} A - A \frac{\partial}{\partial \tau} A^* \right)$$
 (14)

Using this Lagrangian density and the associated Hamiltonian density one finds that the system has the following three invariants

$$I_{1} = \int_{-\infty}^{\infty} dz |A|^{2}$$

$$I_{2} = \frac{1}{2i} \int_{-\infty}^{\infty} dz (A^{*} \frac{\partial}{\partial z} A - A \frac{\partial}{\partial z} A^{*})$$

$$I_{3} = \int_{-\infty}^{\infty} dz [|A|^{2} + \exp(-|A|^{2}) - 1 - |\frac{\partial A}{\partial z}|^{2}]$$

$$I_{3} = \int_{-\infty}^{\infty} dz [|A|^{2} + \exp(-|A|^{2}) - 1 - |\frac{\partial A}{\partial z}|^{2}]$$
(15)

The physical meaning attached to these invariants are: I_1 represents the number of plasmons in the system, I_2 corresponds to the total momentum carried by these plasmons, and I_3 measures the total energy of the system.

The existence of these three invariants should be contrasted with the behavior of known integrable systems^{1,2} such as the Korteweg De Vries equation and the nonlinear Schrodinger equation with cubic nonlinearity, which have been shown^{1,2} to possess an infinite number of invariants.

Due to the strict requirements imposed by the conservation of the infinite set of invariants it is found that such systems support true soliton solutions. The terminology true soliton is used here to emphasize that such entities not only represent nonlinear localized solutions, but in addition their shape, plasmon number, momentum, and energy remain constant after colliding with other such objects. The present investigation is concerned with the properties and formation of soliton-like solutions of Eq.(13). However, it should be realized that these large amplitude entities are not true solitons since, as is shown in Sec. IV C, they can exchange number, momentum, and energy during a collision. Nevertheless, it is found that the magnitude of the exchange of these quantities during a collision is sufficiently small that the terminology large amplitude soliton is quite appropriate in describing the gross behavior of these objects.

III. STEADY STATE BEHAVIOR

In the absence of external pumping (p=0) and in an unbounded medium (c=0) Eq.(13) exhibits steady state soliton-like solutions in which the collapsing effect of the nonlinearity is balanced by the dispersion arising from the electron thermal pressure. These spatially localized solutions have the form

$$A(z,\tau) = a(\xi) \exp[i(kz+\omega\tau)]$$

$$\xi = z - 2k\tau$$
(16)

which transform Eq. (13) into

$$\frac{d^2}{d\xi^2}a + \left[1 - \exp(-a^2) - \overline{\omega}\right] a = 0$$

$$\overline{\omega} = \omega + k^2$$
(17)

For $a^2 << 1$ Eq.(17) reduces in lowest order to the static Schrodinger equation with the cubic nonlinearity often used in describing small amplitude Langmuir solitons. In this small amplitude limit the well-known exact solutions have the form

$$a(\xi) = A_0 \operatorname{sech}(\xi/\Delta)$$

$$\Delta = \sqrt{2}/|A_0|, \ \overline{\omega} = |A_0|^2/2$$
(18)

Since the expressions given in Eq.(18) apply only in the limit $A_0^2 <<1$, it is of interest to determine the properties of the corresponding solutions for arbitrary values of $|A_0|^2$ (e.g., $|A_0|^{2\approx}1.0$) when the full exponential nonlinearity is retained. To extract the large amplitude solutions from Eq.(17) we solve the associated nonlinear eigenvalue problem numerically. The scheme consists of using the Runge-Kutta procedure in which one specifies

 $a = A_G$ and $da/d\xi = 0$ at $\xi = 0$. One then guesses at an initial value for $\overline{\omega}$ and obtains the corresponding wavefunction. This wavefunction is tested for its asymptotic behavior, i.e., whether or not it exhibits an exponential decay for $\xi >> 1$. If the wavefunction does not exhibit the correct asymptotic behavior the previous guess for $\overline{\omega}$ is discarded and a new one is tried. This procedure is found to converge rapidly when implemented on an interactive mathematical computing system.

In Fig. 1 we exhibit the dependence of the nonlinear eigenvalue $\overline{\omega}$ (i.e., the frequency shift) on the peak amplitude of the localized field, $|A_0|$. The solid curve corresponds to the full exponential nonlinearity, and the dashed curve represents the prediction for the cubic nonlinearity, i.e., $\overline{\omega} = |A_0|^2/2$. For the sake of comparison we include in Fig. 1 the broken line curve which corresponds to the eigenvalue that is obtained if in addition to the exponential nonlinearity one retains the second time derivative that is explicitly dropped prior to arriving at Eq.(5). For the steady state case it is a straightforward procedure to keep this contribution.

It is seen from Fig. 1 that the three different approximations give essentially the same result for $|A_0|^2 < 0.2$, thus demarking the regime of applicability of the cubic nonlinearity. For amplitudes such that $|A_0|^2 > 0.5$ it is observed that the saturation of the exponential nonlinearity causes a significant deviation from the overestimate predicted by the cubic nonlinearity. It is of interest to note that the modulational approximation itself produces an underestimate of the actual frequency shift obtained when both the exponential nonlinearity and the full second time derivative are retained. From Fig. 1 it is seen that the modulational representation gives a reasonable approximation (error < 10%) to the exact solution for $|A_0|^2 < 1.2$; for larger values of $|A_0|^2$ the second derivative effect should be retained.

Figure 2 shows the spatial dependence of the nonlinear wavefunctions corresponding to the exponential nonlinearity. In this presentation the wavefunctions are normalized to unity at $\xi = 0$ and the spatial scale used is $|A_0|\xi$, i.e., it contains an explicit amplitude dependence. The reason for using this variable as the abcissa is that it permits the easy identification of the deviation from the solutions to the cubic nonlinearity model given in Eq.(18); these solutions are represented by the innermost curve in Fig. 2. The adjacent curves are the solutions to the exponential problem for the values $|A_0|^2 = 0.25$, 0.50, 1.00, and 2.00, respectively. It is seen from Fig. 2 that the large amplitude Langmuir solitons exhibit a larger spatial extent than is predicted by the cubic nonlinearity. In addition the exponential wavefunctions are more rounded than the sech solutions near $\xi = 0$. Although it is not shown in the presentation of Fig. 2, all wavefunctions are found to approach the sech solution at sufficiently large ξ , as expected.

To quantify the spatial broadening associated with the large amplitude Langmuir solitons we have calculated an effective spatial width δ defined in complete analogy to the width Δ associated with the small amplitude solitons, i.e.,

$$a(\xi=\delta)/|A_0| = \operatorname{sech}(1) \approx 0.65$$
 (19)

The usefulness of this definition becomes apparent when attempting to predict the amplitude of the large amplitude solitons which evolve out of initial standing waves, as is shown in Sec. IV B.

Fig. 3 exhibits the dependence of the effective width δ on $|A_0|^2$. The ordinate is the normalized quantity $\delta |A_0|/\sqrt{2}$. For the cubic nonlinearity this quantity is strictly equal to unity and is represented in Fig. 3 by the dashed line; the continuous curve corresponds to the exponential nonlinearity.

From Fig. 3 one finds that the quantity δ follows quite closely the relationship

$$\frac{\delta |A_0|}{\sqrt{2}} \approx 1 + 0.35 |A_0|^2 \tag{20}$$

which is used later on in Sec. IV B.

The enhanced broadening associated with the saturable nonlinearity should be considered when attempting to compare the relationship between amplitude and width in an experiment involving localized fields. This is particularly relevant if separate independent measurements of width and amplitude are not available and instead one wishes to infer one from the other. The enhanced broadening should also be kept in perspective when attempting to explain the limiting width of localized fields in terms of effective particle damping mechanisms alone.

Having determined the principal steady state properties of large amplitude Langmuir solitons we next proceed to discuss some of the interesting time dependent phenomena associated with these objects.

IV. TIME DEPENDENT PHENOMENA

A. Numerical Method

To investigate the time dependent phenomena we rewrite Eq.(13) in terms of finite differences $(\Delta\tau,\Delta z)$ and use the implicit time-averaged Crank-Nicholson scheme 15 to obtain the matrix equation

$$\alpha_{j} A_{j+1}^{m} + \beta_{j}^{m} A_{j}^{m} + \gamma_{j} A_{j-1}^{m} = w_{j}^{m-1}$$
 (21)

where A_j^m represents the scaled electric field amplitude at time $\tau = m\Delta \tau$ and at the spatial point $z=j\Delta z$. In Eq.(21)

$$\alpha_{j} = \gamma_{j} = -\frac{i\Delta\tau}{2(\Delta z)^{2}}$$

$$\beta_{j}^{m} = 1 + \frac{i\Delta\tau}{(\Delta z)^{2}} - \frac{i\Delta\tau}{2} F_{j}^{m}$$

$$w_{j}^{m+1} = A_{j}^{m-1} - \alpha_{j} (A_{j+1}^{m-1} - 2A_{j}^{m-1} + A_{j-1}^{m-1})$$

$$+ \frac{i\Delta\tau}{2} F_{j}^{m-1} - i\Delta\tau p$$
(22)

where

$$F_{j}^{m-1} = 1 - \exp \left\{ -\left[\left| A_{j}^{m-1} \right|^{2} - c^{m-1} \right] \right\}$$
 (23)

Since Eq.(21) is an implicit relation from which A_j^m is to be determined, one does not know a priori the value of F_j^m which is required to evaluate β_j^m . To overcome this difficulty we use the following linear extrapolation

$$F_{j}^{m} = 1 - \exp \left\{ -\left[\left| 2A_{j}^{m-1} - A_{j}^{m-2} \right|^{2} - c^{m} \right] \right\}$$
 (24)

The spatially independent coefficient c^{m} is determined from the relationship

$$I_{4} = \int_{-2}^{2/2} dz \exp \left\{ -\left[|A_{j}^{m}|^{2} - c^{m} \right] \right\}$$
 (25)

where I_4 is a constant representing the initial number of plasma particles present in the system.

To solve Eq.(21) we utilize the well-known tri-diagonal scheme 16 which provides a simple and fast algorithm for determining the A^{m}_{j} provided that the boundary conditions are known at each time step. Since in the present study we consider both bounded and unbounded systems, it is necessary to use slightly different methods in handling each of these.

In the unbounded case we choose a priori a mesh that is sufficiently long so that the relevant phenomena never have a chance to interact with the numerical boundaries. For this case we set A=0 at these boundaries and legislate that c=0 for all τ , since there is no constraint on particle conservation.

In the bounded case we introduce explicitly a density profile function D(z) which appears through F in the form

$$F = 1 - D \exp \left\{ - \left[\frac{|A|^2 - c}{D} \right] \right\}$$
 (26)

The function D is suitably chosen so that it is essentially unity within the system of interest and falls off sharply to zero near the boundaries. This numerical procedure approximates the situation encountered in a laboratory experiment. To handle the presence of the external pump field, which is the main problem of interest in a bounded system, one needs to consider only the vacuum electrostatic condition A=p at the boundary.

The inclusion of D(z) through Eq.(26) transforms the constraint relation of Eq.(25) into a transcendental equation for c. To obtain c one then uses the method of successive approximations, where the zeroth value is obtained by setting D=1.

The previously described procedure has been implemented with the help of the mathematical on-line computing system at the University of California, Los Angeles.

B. Standing Waves

An interesting and physically important question is: How can a large amplitude Langmuir soliton be generated spontaneously in a plasma device? The answer is that there must be some initial amount of energy supplied to the system and thereafter this energy must be allowed to relax nonlinearly before it is dissipated by collisions or wave-particle interactions. If the initial supply of energy is allowed to expand freely, as is the case in an unbounded system, then the nonlinearity may not be sufficiently strong to create the localization of the field. However, in a bounded system a standing wave pattern is created and can in turn evolve into intense localized fields. Therefore, it is of interest to investigate the time evolution of standing waves in a bounded system.

To study the standing wave problem we consider a plasma slab such that $-50 \le z \le 50$. At $\tau = 0$ the scaled electric field is given by $A(z) = A_0(0) \sin(2\pi z/100)$ with $A_0^2(0) = 0.15$. Fig. 4 shows the spatial dependence of the electric field for various times in the interval $0 \le \tau \le 45$. It is seen from Fig. 4 that early in time the field energy density is relatively small and extends over the length of the system. However, near the peaks of the standing wave the ponderomotive force creates a density cavity (not shown) which begins to localize (or trap) the electric field. This trapping leads to a rapid growth, which saturates at $\tau = 28$. After the sharply localized fields are created they develop nonlinear relaxation oscillations which eventually damp out. In the asymptotic state one obtains large amplitude ($|A|^2 \approx 1.0$)

localized fields whose spatial shape is found, upon detailed examination, to correspond to the wavefunctions of large amplitude Langmuir solitons, which are exhibited in Fig. 2.

The results shown in Fig. 4 demonstrate that the generation of localized electric fields is not unique to the cubic nonlinearity. Furthermore, these results suggest that such entities may be found in plasma devices in which standing waves can be supported.

Figure 5 displays the time evolution of the peak value, $|A_0|^2$, of the electric field associated with the localized entities shown in Fig. 4. It is seen in Fig. 5 that nonlinear relaxation oscillations appear in the process of forming the soliton-like structures, and that the amplitude of these oscillations decreases while their period shortens as the asymptotic state is reached. For the sake of comparison, Fig. 6 exhibits the corresponding behavior for a much smaller initial amplitude of the standing wave $(A_0^2(0)=0.063)$; it should be noted that in Fig. 6 the time scale has been appropriately expanded. It is evident from Fig. 6 that the relaxation oscillations exist also in the small amplitude limit, but their period is much longer than in the large amplitude case.

The asymptotic amplitudes of the soliton-like structures that evolve out of the standing waves can be predicted by using the conservation of plasmon number invariant I_1 given in Eq.(15). The value of this quantity is

$$I_{1} = \int_{-\ell/2}^{\ell/2} dz |A_{0}(\tau=0)|^{2} \sin^{2}kz$$

$$I_{1} = \frac{|A_{0}(0)|^{2}\ell}{2}$$
(27)

where & refers to the length of the system and k to the wavenumber of the initial standing wave. The asymptotic state is given by two soliton-like

structures each having peak amplitude A_0 . According to the small amplitude soliton theory the asymptotic solutions are given by Eq.(18), thus the total plasmon number associated with these structures is

$$I_{1} = 2 \int_{-\infty}^{\infty} dz |A_{0}|^{2} \operatorname{sech}^{2}(\frac{z}{\Delta})$$

$$I_{1} = 4|A_{0}|^{2}\Delta$$
(28)

where the limit of integration has been extended to infinity because the localized fields are far from the boundaries, as is seen in Fig. 4. Using the relationship $\Delta = \sqrt{2}/A_0$

$$|A_0| = \frac{|A_0(0)|^2 \ell}{8\sqrt{2}}$$
 (29)

This prediction can be checked against the result shown in Fig. 6 which corresponds to a lower amplitude soliton (i.e., $|A_0|^2 = 0.25$). For this case $\ell=100$, $A_0^2(0)=0.063$, thus predicting $|A_0|^2 = 0.31$, which is not a bad estimate, since as can be seen from Fig. 6, it agrees with the peak of the relaxation oscillations. However, this prediction does not quite agree with the asymptotic value of $|A_0|^2 = 0.25$. This discrepancy arises due to the enhanced width of the large amplitude solitons, discussed previously in Sec. III, and can be accounted for as will be shown shortly.

The failure of the small amplitude soliton theory becomes apparent when Eq.(29) is used to predict the asymptotic amplitude for the situation shown in Fig. 5. For this case $\ell=100$, $A_0^2(0)=0.15$, thus Eq.(29) predicts $A_0^2=1.76$, which is far off from the observed value of $A_0^2\approx 1.0$ in Fig. 5.

In order to correctly predict the asymptotic value of ${\rm A_0}^2$ one must take into consideration the enhanced width associated with the large

amplitude solitons. This can be done by computing the asymptotic value of I_2 using the large amplitude wavefunctions ψ and their effective width δ

$$I_{1} = 2 \int_{-\infty}^{\infty} dz \psi^{2}(z) \approx 4 |A_{0}|^{2} \int_{0}^{\infty} dz \operatorname{sech}^{2}(\frac{z}{\delta})$$

$$I_{1} \approx 4 |A_{0}|^{2} \delta$$
(30)

where the definition of δ through Eq.(19) proves useful in approximating the value of the required integral. Now equating Eqs.(27) and (30) yields

$$|A_0|^2 \delta = \frac{|A_0(0)|^2 \ell}{8}$$
 (31)

then taking Eq.(20) into consideration results in

$$|A_0|(1+0.35|A_0|^2) = \frac{|A_0(0)|^2\ell}{8\sqrt{2}}$$
 (32)

This large amplitude relationship can be checked against the result of Fig. 5. It predicts the value $|A_0|^2=0.99$, which is represented by the dashed line in Fig. 5, and is shown to be in excellent agreement with the time asymptotic value of $|A_0|^2$.

To correct the previously mentioned discrepancy between the small amplitude soliton theory and the result of Fig. 6 one can use Eq.(32) in the small $|A_0|^2$ approximation, i.e.,

$$|A_0|^{2} \approx \frac{|A_0(0)|^2 \ell}{8\sqrt{2}} (1-0.7|A_0|^2)$$
 (33)

and apply the method of successive approximations to the case of Fig. 6 to yield

$$|A_0|^{2} \approx (0.31) [1-(0.7)(0.31)]$$

$$|A_0|^{2} \approx 0.24$$
(34)

which is represented by the dashed line in Fig. 6, and again is seen to be in excellent agreement with the asymptotic value.

It is worth noting in the context of these results that in a recent laboratory experiment 17 it has been established that density cavities can be produced by the ponderomotive force associated with standing waves. In addition, in another unrelated experiment 18 in a bounded plasma column it has been found by spectroscopic measurements that intense electric fields in the Langmuir frequency range can exist for anomalously long times after the external energy sources have been turned off.

C. Collisions

Although the exponential nonlinearity supports spatially localized solutions which can be generated from initial standing waves, it does not necessarily follow that these objects are true solitons, since for this system only three invariants are known, as is mentioned in Sec. II. To check this point we have investigated the collisional properties of two objects whose initial wavefunctions are given by the steady state solutions discussed in Sec. III, and whose properties are summarized in Figs. 1-3.

The first case we consider is a spatially symmetric collision in which the initial state consists of two large amplitude wavefunctions, each having amplitude $A_0^2(0)=0.4$, centered at $z=\pm 20$, and with scaled wavenumbers $k=\mp 0.4$, respectively. The time evolution of the collision over the interval $0 \le \tau \le 49.2$ is shown in Fig. 7. To the naked eye the series of events depicted in Fig. 7 suggest that if these colliding objects are not true solitons, they are quite close to being so. This behavior is remarkable, considering that at $\tau=24.2$, $|A|^2=1.4$, hence the full saturation of the exponential term is sampled during the collision.

As a check of the numerical procedure it is found that during the entire collision I_1 remains constant to within one part in 10^3 , while the total energy ${\rm I}_{\rm 3}$ varies by less than 0.5% of its pre-collisional value. Because of the symmetry of this problem the momentum I, is expected to vanish identically, and numerically it does so (i.e., a few times 10^{-7}). With this confidence on the numerical procedure we can then probe for quantitative changes in the asymptotic properties of the individual localized fields. It is found for the case of Fig. 7 that the magnitude of the momentum I_2 for each of the individual objects decreases by 4.6%, hence showing that the exponential nonlinearity does cause a distortion during the encounter. The distortion manifests itself by spreading the plasmons away from the peak of the localized fields. Quantitatively the spreading of the tail shows that the plasmon number I, in the region where $|A|^2$ is less than 0.1 of the peak value increases from the pre-collisional level of 4.2% to the final value of 6.6%. This behavior corresponds to the creation of a small uniform background component whose radiation is stimulated by the collision.

Although the symmetric collision is a good test of the overall numerical procedure, a significant amount of quantitative information is unavoidably lost. A better method for obtaining a quantitative test of the distortions produced by the collision is to consider an asymmetric initial state. The outcome of such an encounter is shown in Fig. 8. The initial conditions for this case consist of one large stationary wavefunction (k=0), centered at z=0, and having $|A_0(0)|^2=0.5$. The other smaller wavefunction is centered at z=-25, with $|A_0(0)|^2=0.2$, and has k=0.5, thus moving toward the larger one. Again, the constancy of the invariants was checked, and each was conserved to 0.1% or better.

By examining Fig. 8 in detail it is found that these objects although close to being solitons are not true solitons. The failure manifests itself by a slow motion of the initially stationary object in the direction z<0 after the collision, while the initially moving entity increases its amplitude as it moves toward z>0. Hence, these objects transfer number, momentum, and energy during the collision.

The initially moving object starts out with 36.8% of I_1 , and asymptotically it carries 42.0%, thus demonstrating that plasmons are extracted out of the initially stationary structure. In the process of plasmon transfer the initially moving entity increases its momentum by 2.8% while the stationary object recoils by a corresponding amount. During the collision the energy I_3 of the larger stationary object decreases from 0.19 to 0.15, while that corresponding to the smaller structure increases accordingly. The overall event is analogous to a process in which a small object picks up mass from a larger one.

D. Pumping of a Stationary Soliton

Next we proceed to discuss the response of an initial small amplitude soliton to an external uniform pump electric field. This situation is of interest experimentally since it illustrates the possibility of enhancing the amplitude of soliton structures which may be present in the plasma due to natural causes not directly under the control of the experimentalist. This interaction can thus be used as a test procedure for uncovering the prior existence of a soliton, or alternatively it may be used as a method for generating large amplitude solitons.

The geometry considered consists of a bounded plasma slab whose unperturbed density profile D is shown in Fig. 9. Initially a stationary (k=0) soliton of peak amplitude $A_0^2(0)=0.1$ is located at the center of the slab. At $\tau=0$ the external pump is suddenly turned on and its amplitude is given by p=0.4 exp(i0)

where θ represents the initial relative phase between the soliton and the pump. In general, under typical laboratory conditions θ has a random value, thus it is of interest to illustrate the two possible extreme cases which may be encountered, i.e., $\theta=\pi/4$ and $3\pi/4$. The corresponding results are shown in Fig. 10 for $\tau=1.0$; the initial pattern is included for comparison. From Fig. 10 it is seen that for $\theta=\pi/4$ the pump causes a rapid transfer of energy to the soliton, while for $\theta=3\pi/4$ the pump extracts energy out of the soliton. Since the soliton pumping process is analogous to a driven resonator, eventually the soliton locks to the phase of the pump and the amplitude grows secularly in time regardless of the initial phase; however, this process takes longer than the time used in the display of Fig. 10.

Two additional features present in Fig. 10 worth mentioning are: 1) there is a uniform increase in the amplitude of the electric field over the uniform part of the slab, and 2) there are two narrow spikes which appear at the edge of the slab. The reason for the uniform increase in the electric field is that the entire slab is being pumped near its plasma frequency, hence the effective electric field is enhanced by a factor of ε^{-1} over its vacuum value, where ε refers to the total effective dielectric of the slab (which includes convection effects as well as resonance). The two spikes near the edge of the slab consist of a signal launched by the sharp gradients near the boundaries. This signal behaves as a mode-converted wave which propagates to the interior of the slab due to the second derivative in Eq.(13), which represents the effect of thermal convection.

Figure 11 shows the time evolution of the peak amplitude of the soliton when the external pumping is continued for a longer time (τ =3.0). It is seen in this figure that the amplitude exhibits the characteristic secular growth expected from a resonantly driven oscillator. Accordingly, the soliton

amplitude reaches a rather large value over this interval. For this reason it is more realistic to consider the effects produced by pumping over a short interval such that $|A|^2<1.0$, as is the case for $\tau<1.0$ in Fig. 11.

An interesting procedure which is often employed in laboratory experiments 19 consists of pumping the plasma for a short time and observing the subsequent redistribution of the electric field energy long after the source is turned off. The corresponding situation is shown in Fig. 12, where we exhibit the spatial dependence of $|A|^2$ for various times over the interval $0 < \tau < 20.0$. In this case the pump is started with $\theta = 0$, p = 0.4, and turned off at $\tau = 1.0$. Over this interval the pump increases the original plasmon number I_1 in the system by a factor of 10; this value is found to remain constant after the pump is switched off to within 0.04%.

The sequence of events shown in Fig. 12 indicate that while the pump is on, the amplitude of the initial soliton increases, the mode-converted signals appear near the boundaries, and the uniform field builds-up, as mentioned previously. After the pump is turned off the long scale RF energy which has been injected into the plasma propagates toward the center of the slab where the density cavity generated by the soliton is located. This density cavity then traps a portion of the available energy, and causes the initially small amplitude soliton to evolve into a large amplitude soliton, as is seen at τ =10.0. This large amplitude entity subsequently exhibits relaxation oscillations as it attempts to attain the steady state relationship between amplitude and width discussed in Sec. III. Finally, at τ =20.0 one observes a pattern consisting of a large amplitude soliton and a set of standing waves which are nonlinearly distorted near their peaks. Consequently, this calculation shows how a large amplitude soliton can evolve out of a small amplitude soliton when the latter is stimulated by an external pump for a relatively short time interval.

To complement the spatial evolution shown in Fig. 12 we exhibit in Fig. 13 the time dependence of the peak value of the electric field in the slab. It is seen in Fig. 13 that there is a short interval after turning on the pump $(\tau=0)$ during which the peak amplitude of the soliton is smaller than the pump amplitude, hence the flat line portion of the graph. However, after this short interval elapses one detects the rapid growth of the soliton, represented by the sharp slope following the initial flat portion. The rapid growth of the soliton disappears as soon as the pump is turned off $(\tau=1.0)$. After the pump is removed the soliton peak exhibits a steady but much slower growth resulting from the spatial convergence of the long scale RF energy into the density cavity. Later in time $(\tau-11.0)$ the peak amplitude develops relaxation oscillations which drive the localized field toward the steady state large amplitude solutions discussed in Sec. III.

E. Pumping of Moving Solitons

Having isolated the essential features associated with the external pumping of a stationary soliton (k=0), it is of interest next to extract the corresponding effects associated with moving solitons $(k\neq 0)$. In this case the uniform external pump is capable of increasing the plasmon number and the energy of the soliton, but it does not change the initial nonzero momentum. Therefore it is of interest to inquire as to how the soliton rearranges the surplus of number and energy, e.g., does it break up into a stationary and a moving soliton?

To emphasize the relevant effects associated with the soliton motion we first consider a rapidly moving soliton with wavenumber k=2.0, $|A_0(0)|^2=0.5$, and initially ($\tau=0$) centered at z=20.0. Although this wavenumber leads to a group velocity too large to justify the neglect of ion inertia, it nevertheless

serves to illustrate the effect which is also present for slower solitons. The complex pump amplitude is chosen as p=i(0.07); it is turned on at $\tau=0.5$, and switched off at $\tau=2.5$. The corresponding space-time evolution of $|A|^2$ is shown in Fig. 14 for the interval $0.5 < \tau < 10.0$.

It is seen in Fig. 14 that before the pump is turned on $(\tau < 2.0)$ the soliton motion is as expected from the steady state solutions discussed in Sec. III, i.e., it consists of a pure translation of the smooth envelope with a speed proportional to k; the smooth envelope is shown at $\tau = 0.5$ in Fig. 14. The first observed effect caused by the pump is the modulation of the initial soliton envelope, as is seen at $\tau = 2.5$ in Fig. 14. The envelope acquires a modulational wavelength equal to the initial wavelength of the soliton (i.e., $2\pi/k$). Simultaneously with the development of this spatial modulation it is observed that the peak amplitude of the soliton grows (e.g. $|A_0|^2=1.0$ at $\tau = 5.0$); the plasmon number I_1 increases by 58%. After the pump is turned off it is found that the perturbed soliton does not break into a stationary soliton and a moving soliton. Instead, one observes that a wavepacket continues to move to the right, and as it moves there is a sequential spiking of the electric field. This spiking produces a rippling effect which moves back and forth within the adjacent troughs inside the wavepacket.

Figure 15 exhibits the sequential spiking of the peak electric field associated with the moving wavepacket. It should be noted that the peak value eventually relaxes to a level $|A_0|^2 \approx 1.0$; however, the oscillations do not disappear over the time observation. This behavior should be contrasted to that of the stationary soliton, shown in Figs. 12, 13. The external pumping of a stationary soliton is capable of generating a large amplitude soliton, i.e., it transforms a small soliton into a large one. This transformation is accomplished through relaxation oscillations which disappear quickly. In the

rapidly moving soliton case the pumping does not create a large amplitude moving soliton, (i.e., one of the solutions of Sec. III) at least not over the time of observation. Instead, the pump excites the periodic spiking of the wavepacket which persists for a long time.

Figure 16 exhibits the effect of pumping on a slowly moving soliton having initial values k=0.2, $|A_0(0)|^2=0.5$, and centered at z=20.0. The pump behavior is chosen as in the rapidly moving case described previously. It is seen in Fig. 16 that in this case the small amplitude soliton increases its amplitude and experiences a small distortion in its initially smooth envelope. The reason for the smaller distortion of the envelope is that the wavelength in this case is much longer than that used in obtaining Fig. 14. The back and forth rippling of the wavepacket is hardly noticeable in Fig. 16; however, the associated spiking of the peak amplitude is evident in the oscillations shown in Fig. 17. Again, it is observed in Fig. 17 that the maximum amplitude saturates at a level $|A_0|^2\approx1.0$, but the oscillations do not disappear over the observation time ($\tau<35.0$). It is worth noting that in this case the plasmon number is increased by 77%, thus indicating a better coupling to the external pump than is provided by the faster moving soliton.

F. Pumping of Colliding Solitons

To complete the survey of soliton behavior in the presence of an external pump, we next consider the effects produced by pumping while two solitons collide with each other. The case presented corresponds to the same initial conditions used in the asymmetric collision previously discussed in Sec. IV B.

Figure 18 displays the evolution of the collision over the interval $16.0 \le \tau \le 54.0$. The behavior for $\tau < 16.0$ is not shown; during this interval the smaller soliton propagates toward the larger one, which is initially stationary

and centered at z=0. The external pump is turned on at τ =16.0 with p=0.005 and is switched off at τ =32.0, just as the solitons begin to separate from each other, i.e., pumping occurs while the solitons are interacting strongly with each other.

It is seen from Fig. 18 that there are no major qualitative disruptions in the scattering behavior already discussed in Sec. IV B. The most striking change is associated with the distortion induced on the smaller moving soliton. This distortion is seen in Fig. 18 in the interval 32.0 < \tau < 54.0 and consists of the sequential spiking of the electric field, which was previously identified in Sec. IV E as the dominant effect associated with the pumping of a single moving soliton. The distortion manifests itself through amplitude oscillations analogous to those shown in Fig. 17. Accordingly, the behavior observed is quite similar to what one would obtain if a single noninteracting soliton were considered. This behavior is remarkable considering that most of the pumping occurs while the two solitons are interacting strongly.

Quantitatively, it is found that the external pumping increases the total plasmon number I_1 by 14% of the initial value; 68% of this increase is transferred to the smaller moving soliton. Interestingly, it is found that the pumping decreases the momentum I_2 of the smaller moving soliton by 4.3%. This result is to be contrasted with the collision without pumping in which the moving soliton increases its momentum by 2.8%. The total momentum of the system remains constant, as expected.

V. CONCLUSIONS

The present study has isolated some of the experimentally interesting properties of the large amplitude localized electric fields in the Langmuir frequency range which are described by the nonlinear Schrodinger equation with the full exponential nonlinearity retained. The steady state eigenvalues and the associated wavefunctions have been calculated and compared against the well-known small amplitude soliton solutions. The principal differences found are that the large amplitude solitons exhibit a larger spatial extent and a smaller nonlinear frequency shift than is predicted from the small amplitude theory.

The time dependent behavior of the large amplitude solitons has been investigated in a bounded plasma slab. It is found that initial standing waves evolve into the large amplitude steady state solutions while exhibiting nonlinear relaxation oscillations. The observed amplitude of the time asymptotic state has been shown to be in excellent agreement with the calculated values which use the nonlinear relationship between the width of the steady state large amplitude solutions and their peak amplitude; the small amplitude soliton theory is found to significantly overestimate the asymptotic amplitudes.

The collisional behavior of the large amplitude solutions of the exponential nonlinearity has been examined for both symmetric and asymmetric cases. The striking feature is that these objects behave almost like true solitons even though during the collision the saturation effect of the exponential nonlinearity is sampled. The magnitude of the deviation from true soliton behavior as gauged by the individual asymptotic changes in number, momentum, and energy conservation is found to be less than 5%. Thus the terminology large amplitude Langmuir soliton is appropriate when referring to the gross behavior of these entities.

The external pumping of stationary, moving, and colliding large amplitude solitons has been investigated. It is found that an initial stationary small amplitude soliton can evolve into one of the stationary large amplitude solutions after undergoing relaxation oscillations. A moving soliton is found to develop a new type of nonlinear distortion which results in the periodic spiking of the electric field inside the moving wavepacket. It is found that by pumping during a collision no major distortions develop, and that the changes produced are similar to those induced on individual noninteracting solitons.

Although the underlying model used in the present study to investigate the properties of large amplitude Langmuir solitons neglects several important effects (e.g., ion inertia, damping) it is expected that many of the general features uncovered may carry over when these other interactions are included, in particular when external pumping is present.

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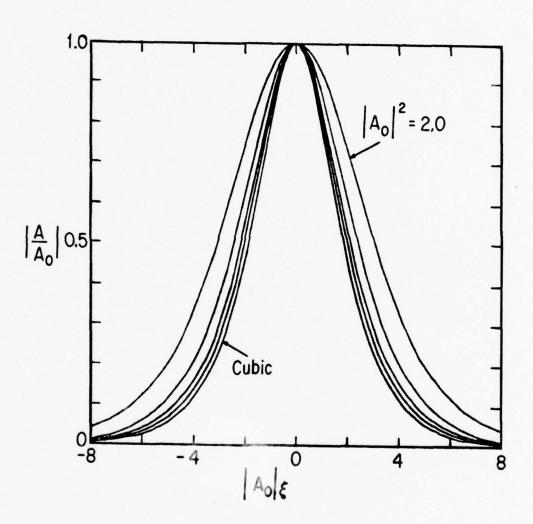
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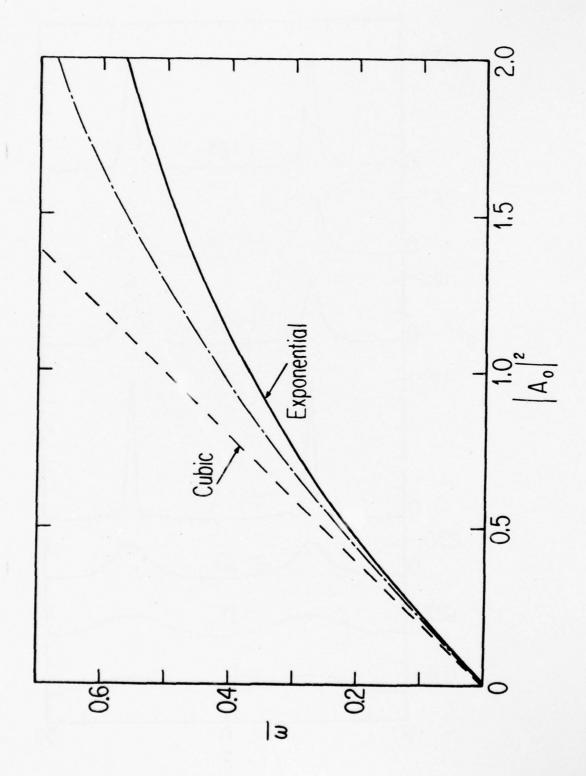
FIGURE CAPTIONS

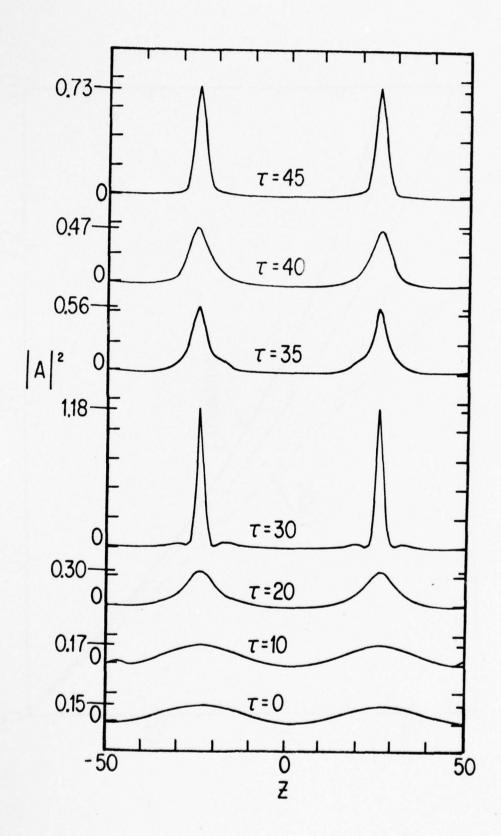
- FIG. 1 Amplitude dependence of the scaled nonlinear frequency shift $\bar{\omega}$. The solid curve corresponds to the exponential nonlinearity; the dashed curve to the cubic nonlinearity. The broken line is the result obtained if the second derivative is retained in addition to the exponential nonlinearity.
- FIG. 2 Spatial dependence of the normalized large amplitude wavefunctions. The innermost curve represents the family of sech solutions (cubic nonlinearity) while the adjacent curves are obtained for $|A_0|^2=0.25$, 0.50, 1.00, and 2.00, respectively. Note the amplitude dependence of the abcissa.
- FIG. 3 Amplitude dependence of the effective spatial width δ corresponding to the steady state solutions with the full exponential nonlinearity retained.
- FIG. 4 Space-time evolution of initial standing waves into large amplitude solitons.
- FIG. 5 Time dependence of the peak amplitude of the localized electric fields generated by the standing waves of Fig. 5. The dashed line corresponds to the analytic prediction which uses the effective width of the large amplitude wavefunctions.
- FIG. 6 Time dependence of the peak amplitude of the localized electric fields generated by a standing wave having initial amplitude $|A_0(0)|^2=0.063$. The small amplitude soliton theory predicts a time asymptotic value of 0.31. The dashed line corresponds to the large amplitude theory.

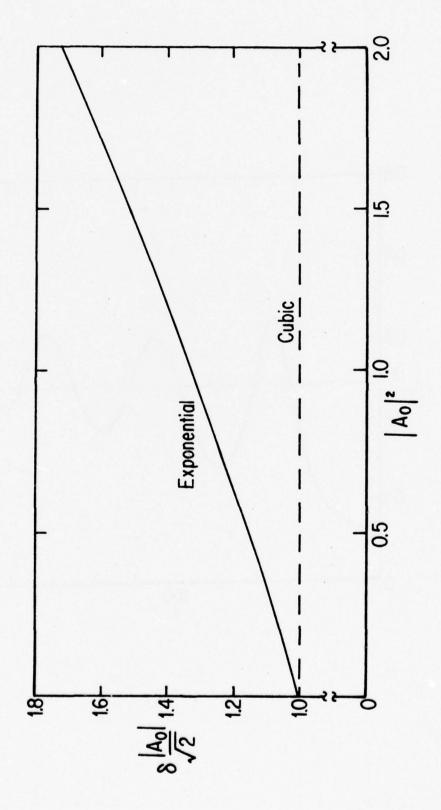
- FIG. 7 Space-time evolution of the symmetric collision between two large amplitude solutions initially centered at $z=\pm 20$ with k=-0.4.
- FIG. 8 Space-time evolution of the asymmetric collision between two initial large amplitude solutions. The larger object is initially stationary (k=0) and centered at z=0. The smaller moving soliton is initially centered at z=-25 with k=0.5.
- FIG. 9 Unperturbed density profile D used in the calculation of the pumping of a stationary soliton.
- FIG. 10 Spatial dependence of the total electric field after pumping with $\theta=\pi/4$ and $3\pi/4$ for $\tau=1.0$. The initial soliton $(\tau=0)$ is included.
- FIG. 11 Time evolution of the peak amplitude of the total electric field in the plasma slab during pumping.
- FIG. 12 Space-time evolution of the pumping of an initial small amplitude soliton located at the center of the plasma slab. The pump is turned on at $\tau=0$ with p=0.4, and turned off at $\tau=1.0$.
- FIG. 13 Time dependence of the peak amplitude of the total electric field in the plasma slab corresponding to the evolution shown in Fig. 12.
- FIG. 14 Effect of external pumping on a rapidly moving soliton initially centered at z=-20, with k=2.0 and $|A_0(0)|^2=0.5$. The pump amplitude is taken as p=i(0.07); it is turned on at $\tau=0.5$ and turned off at $\tau=2.5$.

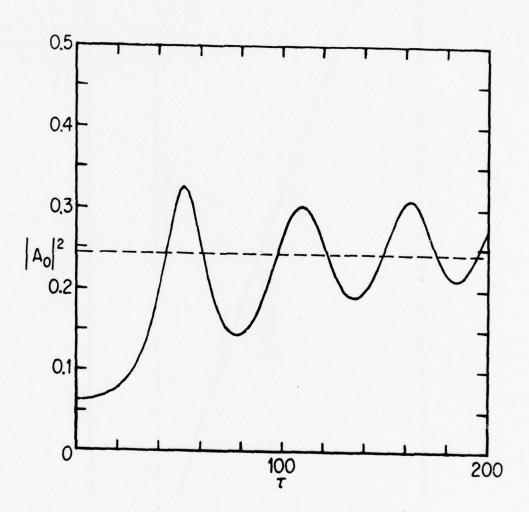
- FIG. 15 Time dependence of the peak amplitude of a rapidly moving soliton after being subjected to external pumping, as shown in Fig. 14. The pump is turned on at $\tau=0.5$ and turned off at $\tau=2.5$.
- FIG. 16 Effect of external pumping on a slowly moving soliton initially centered at z=-20, with k=0.2 and $|A_0(0)|^2$ =0.5. The pump amplitude is taken as p=i(0.07); it is turned on at τ =0.5, and is turned off at τ =2.5.
- FIG. 17 Time dependence of the peak amplitude of a slowly moving soliton after being subjected to external pumping, as shown in Fig. 16. The pump is turned on at τ =0.5 and turned off at τ =2.5.
- FIG. 18 Effect of external pumping on a large amplitude soliton collision. The collision in the absence of pumping is shown in Fig. 8. The pump is taken as p=0.005; it is turned on at τ =16.0, and is turned off at τ =32.0.

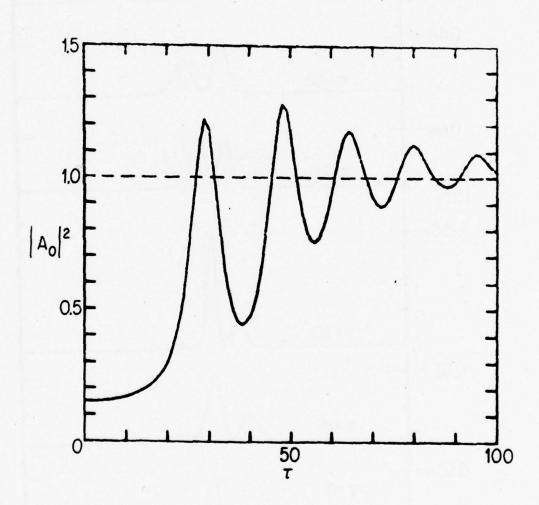


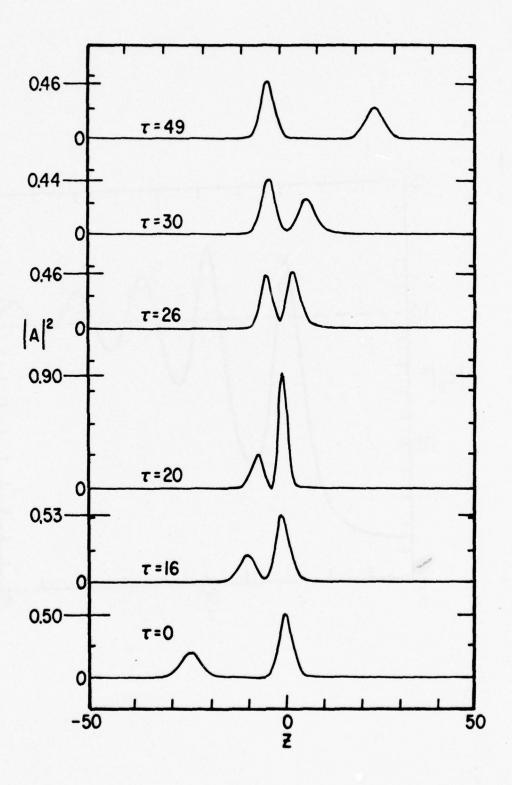


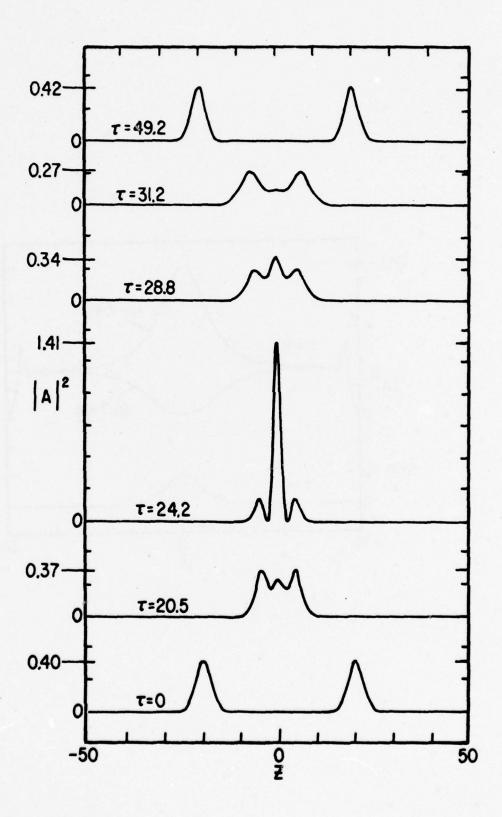


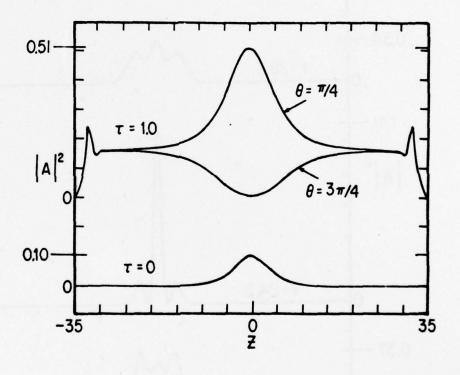


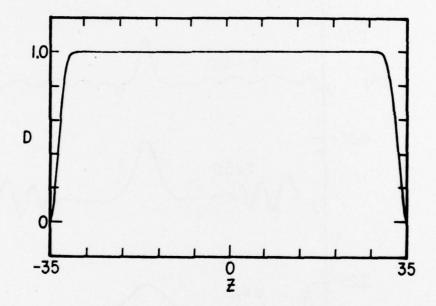


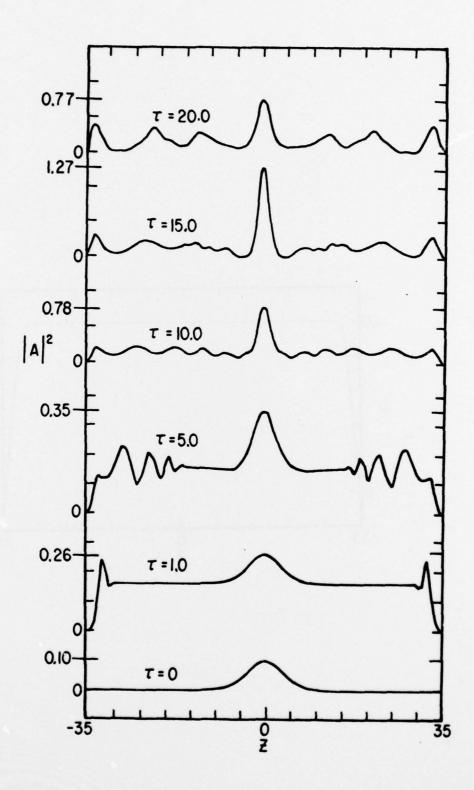


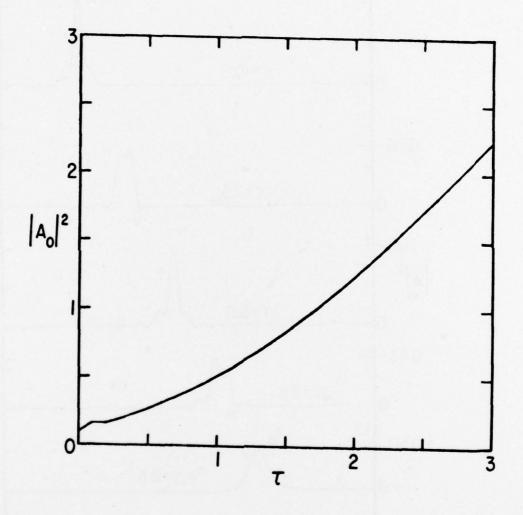


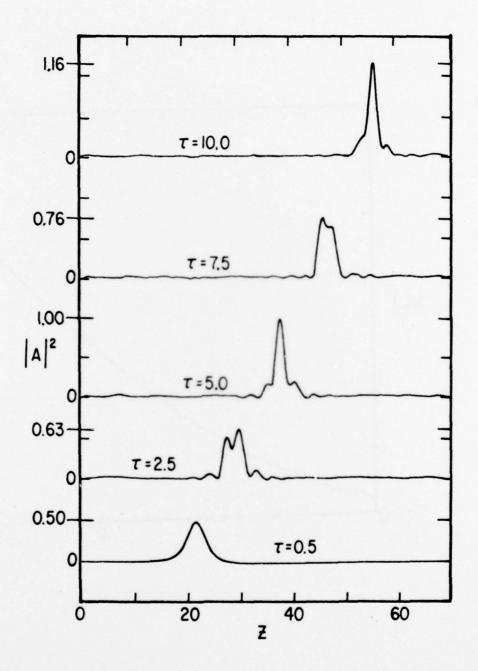


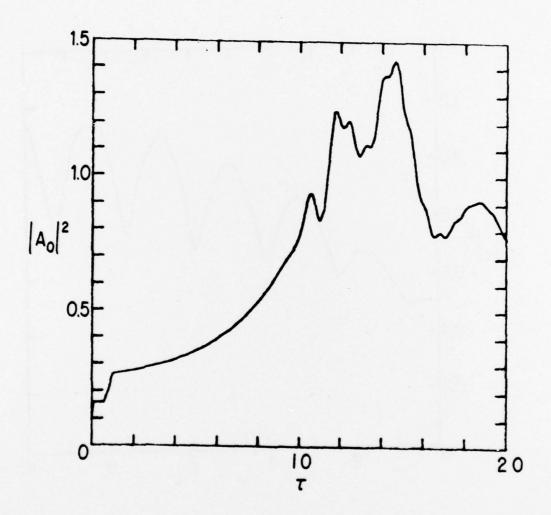


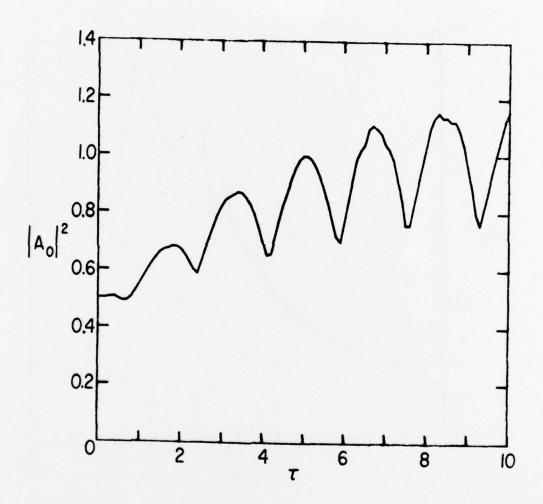


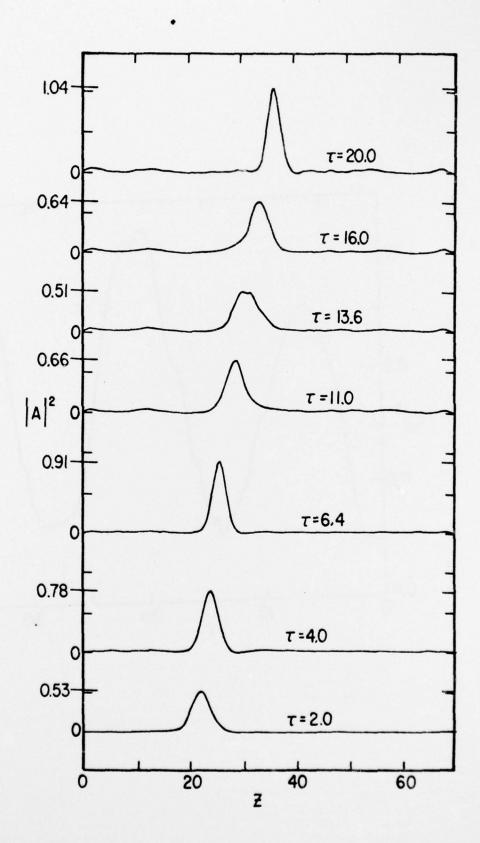


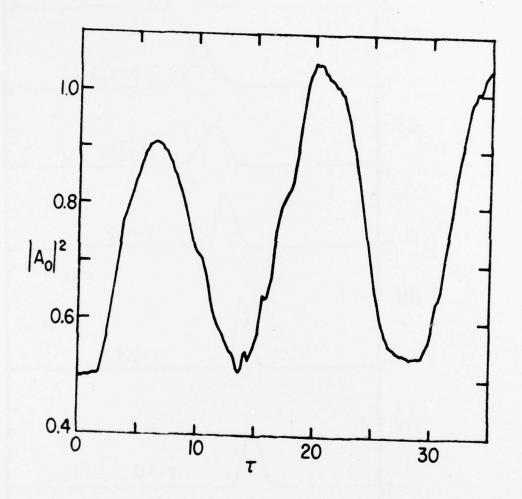


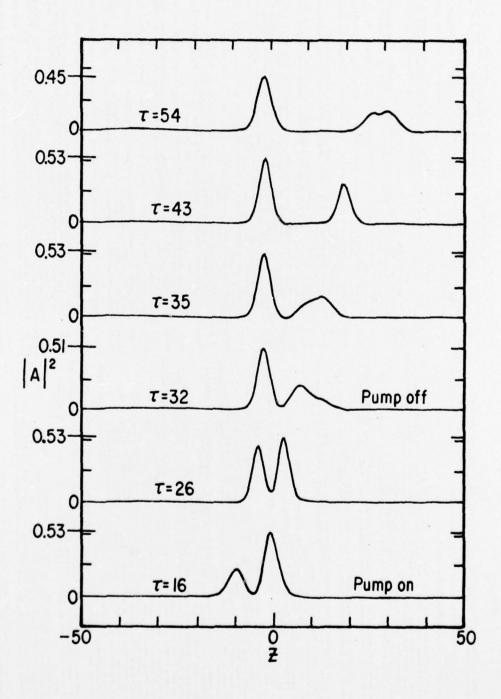












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